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EVALUATION OF MODES IN DIELECTRIC RESONATORS USING A SURFACE INTEGRAL EQUATION FORMULATION

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Summary

A moment method solution procedure for rotationally symmetric dielectric bodies has been applied to isolated cylindrical dielectric resonators, and the frequencies as well as Q factors due to radiation have been determined for several of the lowest modes including those of hybrid type.

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The advantages of dielectric resonators are their small size, low cost, and good temperature stability. One of the important disadvantages is a proximity of resonant frequencies of various modes. It is therefore of great practical importance to know the resonant frequency and the field pattern not only for the desired mode of operation (usually $TE_{01\delta}$) but also for other, undesired modes. The numerical procedure reported here is capable of providing accurate values of resonant frequencies and Q factors for all the modes of interest.

Exact field solutions for dielectric resonators are presently available only for the modes with no azimuthal variation (first subscript m=0), and for resonators which conform to a cylindrical system of coordinates [1] to [4]. The higher modes ($m\neq 0$) have been included in a study of scattering from rotationally symmetric bodies by Barber, et al. [5]. However, their procedure, which employs the extended boundary condition method, has not yet been applied to the study of dielectric resonators. In this paper we utilize the method of moments for the analysis of dielectric resonators. The method is applicable for dielectric bodies of revolution with arbitrary cross section and for any azimuthal variation (including hybrid modes with $m\neq 0$). Our approach is based on the solution of a surface integral equation. It offers several computational advantages over finite difference equation or volume integral equation approaches, particularly when the resonator is not enclosed in a metal boundary, such as in the case of isolated resonators.

The surface integral equation for bodies of revolution has been formulated in [6]. The scattering problem of a rotational body illuminated by an incident field is shown in Fig. 1. The continuity of the electric and magnetic fields tangential to the surface requires

$$-[\overline{E}(\overline{J},\overline{M}) + \overline{E}^{s}(\overline{J},\overline{M})]_{tan} = \overline{E}^{inc}_{tan} \quad (1)$$

$$-[\overline{H}(\overline{J},\overline{M}) + \overline{H}^{s}(\overline{J},\overline{M})]_{tan} = \overline{H}^{inc}_{tan}, \quad (2)$$

where $\overline{J}$ and $\overline{M}$ are equivalent electric and magnetic surface currents. The electric and magnetic fields on the left-hand side are next expressed in terms of the electric and magnetic vector and scalar potentials. The rotational symmetry of the body is utilized by expanding all the fields and surface currents in Fourier series in ϕ. For example

$$\overline{E}^{inc}(t,\phi) = \sum_{m=-\infty}^{\infty} \overline{E}^{inc}_{m}(t)e^{jm\phi} \quad (3)$$

Application of the method of moments then yields a set of simultaneous equations which may be represented in matrix form for each Fourier component as

$$\underset{\sim}{Z}|I\rangle = |V\rangle \quad (4)$$

where $\underset{\sim}{Z}$ is the moment matrix, $|V\rangle$ is the forcing vector, and $|I\rangle$ is the column vector containing the surface current coefficients to be determined. Since there are two currents (electric and magnetic), and two vector components (along t and along ϕ) of each current, in total there are approximately four unknown surface current coefficients for each point used to define the generating arc of the body of revolution.

The individual resonant modes are found by solving the homogeneous system (eq. (4), with $|V\rangle = |0\rangle$) for a particular Fourier component m. Thus the determinant of the moment matrix $\underset{\sim}{Z}$ must be zero:

$$\det(\underset{\sim}{Z}) = 0 \quad (5)$$

The complex roots of (5) are

$$s_n = \delta_n + j\omega_n \quad (6)$$

In the above, ω_n is the resonant frequency of the mode (m,n) and δ_n is inversely proportional to the radiation Q factor:

$$Q_n = \frac{\omega_n}{2\delta_n} \quad (7)$$

In Fig. 2, the relative value of the determinant (for m=0) is plotted for a cylindrical dielectric resonator of radius a=5 mm, length h=5 mm and dielectric constant $\varepsilon_r=35$. The generating arc of the body of revolution is described by 7 points ($\underset{\sim}{Z}$ is a 22 × 22 matrix), and the complex frequency s_n in (6) is purely imaginary. In the range between 2 GHz and 8 GHz two distinct minima of the absolute value of the determinant are visible, one at 5.1 GHz and the other at 7.6 GHz. With the use of diagrams from Gelin et al. [4], the two modes can be identified as $TE_{01\delta}$ and $TM_{01\delta}$.

More accurate values of the resonant frequencies as well as the values of the corresponding Q factors can next be determined by extending the search for roots to the complex frequency plane. It has been found that, by utilizing separately the real and imaginary parts of the determinant, a simple linear search procedure is possible. Therefore, it was decided to use this type of search procedure for finding the exact position of roots on the complex plane. Each step in the linear search requires only 3 points, and no more than three consecutive steps have been necessary. The numerical convergence was studied by increasing the number of points N on the generating arc from 7 (22 unknowns) to 25 (94 unknowns). The results for the $TE_{01\delta}$ mode are shown in Table I. It is seen that the radiation Q factor is more sensitive to the number of unknowns than the resonant frequency. When f and Q from Table I are plotted vs. 1/N, one can linearly extrapolate the values to the case $N\to\infty$. These limit values are presented in the last column of Table I. Agreement of the frequency with reference [4] is better than 1%, while the extrapolated Q factor is lower, coming closer to the values given by [7].

The power of our numerical technique is demonstrated in Fig. 3. By simply changing the input parameter to m=1, the same resonator as before yields the determinant values such as shown in the figure. The modes are now hybrid. By observing the minima of the absolute value, one can clearly identify two resonances, one at 6.3 GHz and another at 7.1 GHz, belonging to modes $HEM_{11\delta}$ and $HEM_{12\delta}$.

The example which follows is computed for a JFD resonator, type DRD 105 UD 046, with ε_r=38, h=4.6 mm and a=5.25 mm. The number of points used to model the contour is N=13, and the computed values of f and Q are listed in Table II for the lowest four modes.

The experimental verification of the resonant frequency and the Q factor was performed with a network anlyzer, by using the transmission method. The resonator was situated in a box padded with absorbing material. The results of measurement are also shown in Table II. The agreement in resonant frequency is about 1%, while the agreement in Q factor is about 20%, except for the mode $HEM_{21\delta}$. As the transmission method is not very reliable for the Q measurement, the reflection method was also attempted. It proved to be difficult to obtain sufficient coupling to the coaxial line, especially for the modes with low Q factor. For the two modes where the reflection measurement was possible, the agreement with computed values was good, as can be seen from the last column in Table II.

Encouraged with this experimental verification of the numerical procedure, we computed the universal mode chart for cylindrical dielectric resonators with ε_r=38. The chart is shown in Fig. 4, displaying the value of $k_o a$ vs. the ratio a/h. In order to economize the computer time, the resonant frequencies were determined by simply observing the minimum of the determinant on the imaginary axis of the complex plane. The minimum number of points modeling the cylinder was N=13, but it was necessary to increase it for certain values of a/h. Such changes of N are the reason for slight kinks on the curves shown.

It is seen that for the range a/h between 0.3 and 3, the mode $TE_{01\delta}$ is the dominant mode. Also, it is noteworthy to observe that the resonant frequencies of the modes $TM_{01\delta}$ and $HEM_{21\delta}$ are running inconveniently close to each other over a wide range of values a/h. The classification of modes in the figure is done by comparing the resonant frequencies with the eigenvalues of the dielectric rod waveguide [8]. Admittedly, this is an approximate identification technique, and the second subscript of some modes may be in error. In the future, it is planned to compute the local field distribution in the resonator at each of the complex resonant frequencies. Such a detailed field distribution will hopefully enable us to make a positive mode identification.

References

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Table I

Convergence of results for $TE_{01\delta}$ mode (ε_r=35)

N	7	13	19	25	∞
f	5.0863	5.0931	5.1027	5.1077	5.124
Q	73.76	45.19	41.37	39.78	34.0

Table II

Comparison of computed and measured results (ε_r=38)

Mode	Computed		Measured		
	f(GHz)	Q	f(GHz)	Q(transm.)	Q(refl.)
$TE_{01\delta}$	4.82	48.5	4.85	51	47
$HEM_{12\delta}$	6.63	51.9	6.64	64	--
$TM_{01\delta}$	7.51	77.0	7.60	86	--
$HEM_{21\delta}$	7.75	291.0	7.81	204	288

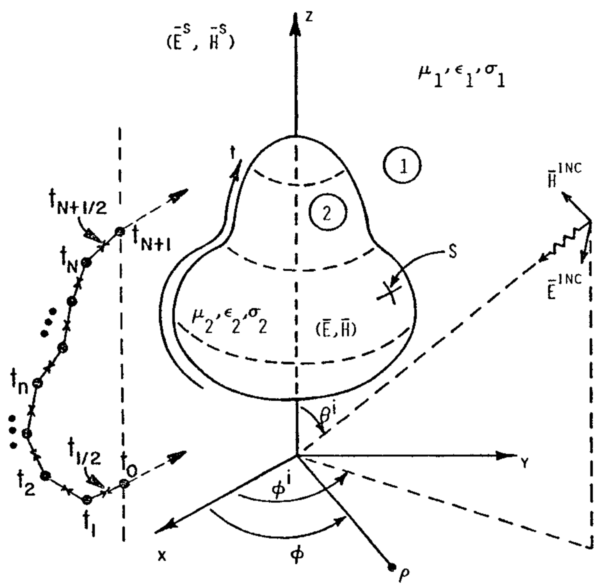

Fig. 1 Rotationally Symmetric Body Illuminated By An Incident Plane Wave.

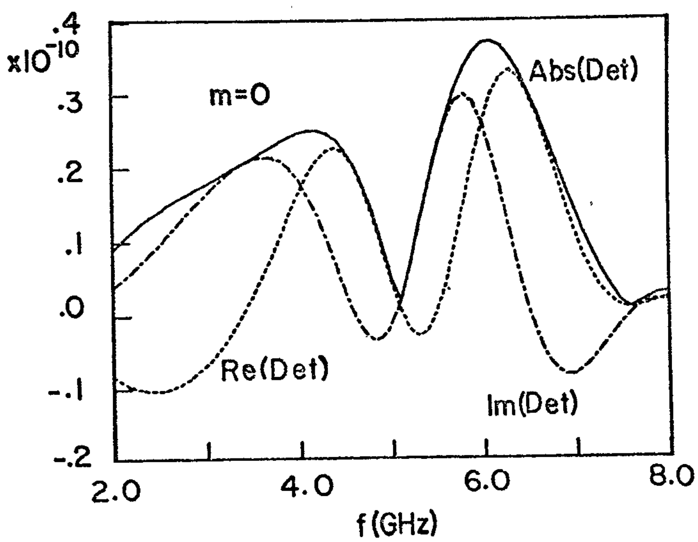

Fig. 2 Determinant vs. Frequency, m=0.

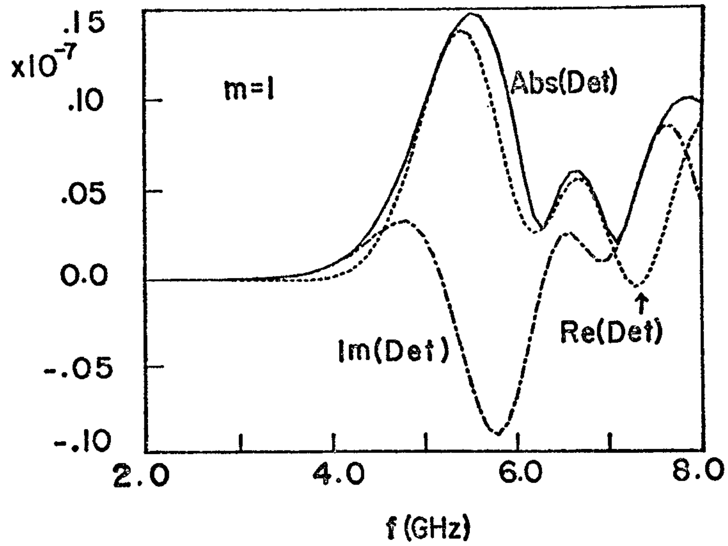

Fig. 3 Determinant vs. Frequency, m=1.

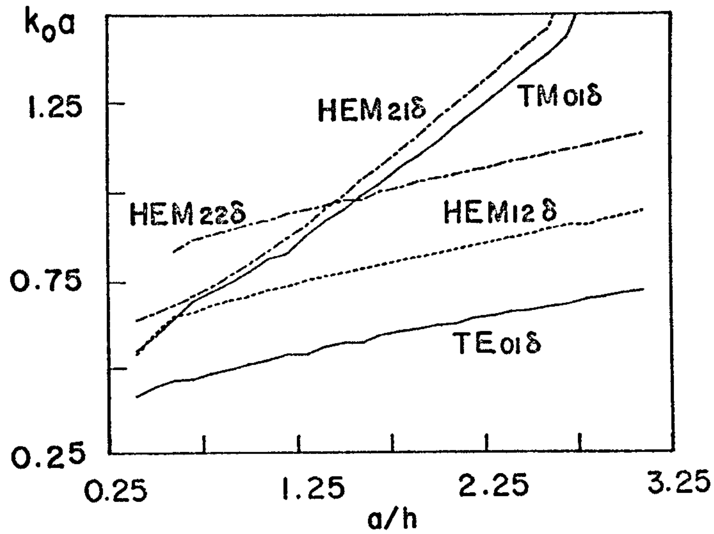

Fig. 4 Universal Mode Chart For Isolated Cylindrical Dielectric Resonators, ε_r=38.